Discretisation and Continuity: Simulating the Emergence of Symbols in Communication Games

Robert Lieck (research@robert-lieck.com)
Leona Wall (research@leona-wall.com)
Martin Rohrmeier (martin.rohrmeier@epfl.ch)

Digital and Cognitive Musicology Lab,
École Polytechnique Fédérale de Lausanne
1015 Lausanne, Switzerland

Abstract

Signalling systems of various species (humans and non-human animals) as well as our world both exhibit discrete and continuous properties. However, continuous meanings are not always expressed using continuous forms but instead frequently categorised into discrete symbols. While discrete symbols are ubiquitous in communication, the emergence of discretisation from a continuous form space is not well understood. We investigate the emergence of discrete symbols by simulating the learning process of two agents that acquire a shared signalling system. The task is formalised as a reinforcement learning problem in continuous form and meaning space. We identify two central causes for the emergence of discretisation: 1) suboptimal signalling conventions and 2) a topological mismatch between form and meaning space. A long version of this paper has been accepted for publication in *Cognition* (International Journal of Cognitive Science).

Keywords: communication games; reinforcement learning; language evolution; symbol grounding; discretisation

Introduction

Utterances of humans and various non-human animals exhibit both discrete and continuous properties (Wilbrecht & Nottebohm, 2003; Janik & Slater, 1997; Rendell & Whitehead, 2001; Kuhl, 2004; Ouattara et al., 2009). They are composed of one or more smaller acoustical units (e.g. a sentence composed of words and words composed of phonemes), each having additional continuous features (e.g. loudness, pitch, duration and timbre). The discrete components can be used to denote various things and circumstances in the world, such as objects or the presence of particular predators (Ouattara et al., 2009). However, discrete symbols[1] can only approximately represent continuous information, such as shades of a colour or gradual changes in emotion. The continuous component, on the other hand, allows for a nuanced representation of gradual aspects, such as the emotional state of the sender (Scherer, 2003; Dimos et al., 2015). In contrast, it does not provide absolute certainty about discrete aspects, such as whether or not a predator is approaching.

Interestingly, we frequently observe discretisation in parts of the world that are inherently continuous. In human language, for instance, the colour spectrum is mapped to distinct colour words (Steels & Belpaeme, 2005; Roberson et al., 2004; Sandhofer & Smith, 1999). Discrete entities are omnipresent in communication (words, characters, symbols etc.) and we are used to employing them to describe continuous aspects of the world: "Bricks are primarily red, with a hint of orange and a little bit of grey." Composing discrete units might therefore seem like an obvious approach for communicating continuous meanings. From an evolutionary perspective, however, it seems much more natural and effective to use continuous properties of the form to communicate continuous aspects of the world. Such as when communicating just *how* different something is by varying the way we pronounce the word "very" when saying: "It's veeerrry different."

> *So why are continuous meanings not always communicated using continuous forms?*

The goal of this paper is to understand what causes a single connected region in continuous meaning space to be split up and expressed using multiple discrete symbols. To investigate such an early evolution of communication, potentially even prior to the development of complex phonological and syntactic structures, we simulate the learning process of two agents (any human, non-human animal, machine or any other entity that engages in an act of communication) that acquire a shared signalling system. We employ a setup that has been used in various other related works, with the important difference that we do not assume the existence of discrete symbols or categories at any point and all steps operate in continuous space. This allows for the development of entirely continuous signalling conventions, based on which we then empirically identify the emergence of discretisation. To the best of our knowledge, our work represents the first attempt to explain the emergence of discrete symbols in regions with continuous meanings based on simulations from first principles.

Symbols

When it comes to the difference between discretisation and continuity, a central problem in the description of real-world signalling systems is that, in a way, they are discrete and continuous *at the same time*. Whether a particular system can be considered discrete or continuous usually depends on the level of description. Should the dynamics within a digital computer be considered discrete (because it operates on 0s and 1s) or continuous (because the 0s and 1s are effectively

[1] *Symbols*, as the term is used in this paper, may have both a discrete and a continuous character. To denote physical quantities, signals or objects that carry information and may be used in communication, we speak of *forms*, in the sense the term is used in semiotics for the physical appearance of a sign (Chandler, 2017).

represented by continuous voltages)? Should the words of a language be considered discrete entities (because they can be listed, one by one, in a dictionary) or continuous (because each word can be pronounced in infinitely many different ways)? We believe that mistaking a difference in the level of description for the fundamental question of whether something is "truly" discrete or continuous is one of the most common reasons for confusion. The notion of symbols we put forward in this paper therefore comprises both aspects and can be defined as follows:

> *A symbol is a connected region in form space, in which all forms can be effectively used in communication and that is separated from other symbols.*

This definition has two major advantages that we rely on in our evaluation of the experiments: 1) it allows for empirically detecting symbols in communication data with a continuous form space by identifying a surrounding margin of unused forms and 2) it is compatible with continuous (iconic)[2] mappings of the region in form space that is covered by a specific symbol. Moreover, this definition of symbols integrates well with the notion of signs in semiotics, which represent a link between forms and meanings that cannot be detached from their concrete usage in communication (Chandler, 2017).

Related Work

A synthetic approach to language evolution has already been used in communication games (Wittgenstein, 1953; Steels, 1997; Nolfi & Mirolli, 2010) to study various properties of language and language evolution via robotic experiments and computer simulations of communicating agents (Oliphant & Batali, 1997; Cangelosi & Parisi, 2002; Christiansen & Kirby, 2003). Existing research has mainly focused on one of three distinct tasks: **1) Syntax and Semantics:** How do the rules for combining symbols (syntax) can be learned and how do the resulting symbol combinations acquire meaning (semantics) (Steels, 1997; De Jong, 1999; Nowak & Krakauer, 1999; Steels & Belpaeme, 2005; Oudeyer & Kaplan, 2007; Bleys et al., 2009; Spranger, 2016). These approaches conceptually stay within the purely discrete realm. **2) Vocalisation:** Learning a mapping from discrete symbols to a continuous signal that is transmitted between the agents (*imitation games,* De Boer, 2000; Oudeyer, 2005; Moulin-Frier & Oudeyer, 2012; Moulin-Frier et al., 2014; Murakami et al., 2015). Here the existence of discrete symbols is presupposed. **3) Emotional Speech Synthesis:** Expression of emotions in speech synthesis by appropriately shaping the continuous representation (e.g. pitch and duration) of the discrete symbols, again, assuming their existence (Oudeyer, 2003; Schröder, 2001).

The closest precursor to our work is the one by Zuidema & Westermann (2003). While working in an entirely discrete setup, they introduce a continuous topology in form and meaning space by means of a noisy transmission distribution and a reward/value function. Technically, our sender policy, receiver policy, transmission distribution, and reward function (see below) are the continuous equivalent of their S, R, U, and V matrices.

[2] Continuous mappings are *iconic* in that the form and the meaning space match in terms of their topology and a continuous variation of the form corresponds to an analogous variation in meaning (also cf. De Boer & Verhoef, 2012; Chandler, 2017).

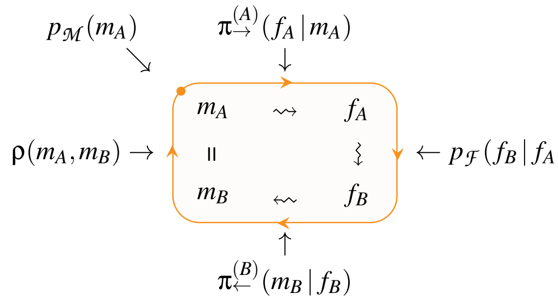


Figure 1: Communication cycle from A (sender) to B (receiver): a meaning $m_A \in \mathcal{M}$, corresponding to A's perception of the world, is sampled from the meaning distribution $p_{\mathcal{M}}$; A chooses a form $f_A \in \mathcal{F}$ to communicate m_A according to its sender policy $\pi^{(A)}_{\rightarrow}$; this form is transmitted as $f_B \in \mathcal{F}$ to B via the transmission distribution $p_{\mathcal{F}}$; B interprets the form f_B to have the meaning $m_B \in \mathcal{M}$ according to its receiver policy $\pi^{(B)}_{\leftarrow}$; finally, the success of communication is evaluated by comparing m_A and m_B and both agents receive a reward according to the reward function ρ.

Our experiments generalise their setup in that we 1) work in an entirely continuous setting, 2) only use the reward feedback of single communication acts for learning (they present preliminary results for this scenario with "limited feedback"), and 3) investigate the effect of different topologies, in particular, of a topological mismatch between form and meaning space.

The interplay of continuous mappings and discretisation in the case of a topological mismatch between the form and meaning space was empirically investigated in humans by Little et al. (2017). The case of a topological mismatch is also discussed by De Boer (2012) and De Boer & Verhoef (2012). They note that a continuous (iconic) mapping is not possible in that case but restrict their considerations to the case where the form space has a lower dimensionality than the meaning space (1D versus 2D in their experiments).

Methods

We simulate communication games, in which two agents repeatedly engage in communication by exchanging forms and receive feedback about the quality of their communication to improve their policies. A complete communication cycle is illustrated in Figure 1. We use a similar setup as in previous works (esp. Zuidema & Westermann, 2003; De Boer & Verhoef, 2012), with the important difference that we generalise to the fully continuous setting. We perform several experiments with different topologies of the form and meaning space, showing that discrete symbols embedded into these continuous spaces emerge as a result of the learning process.

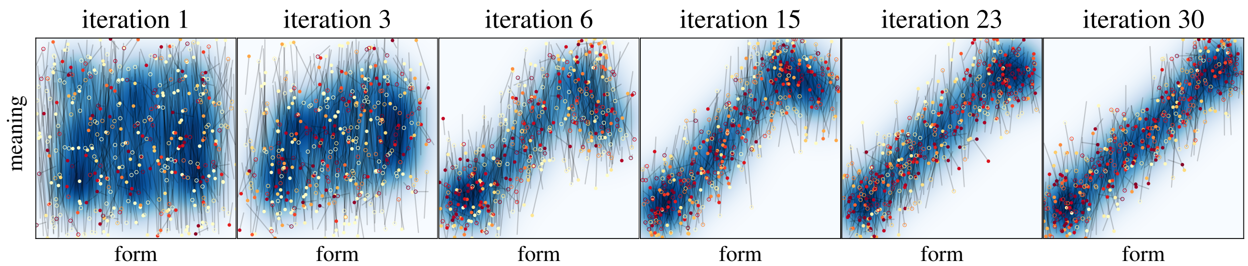
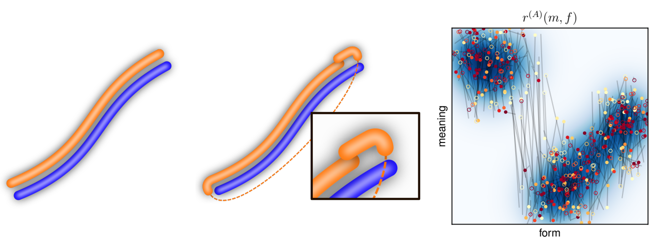


Figure 2: Learning progress in the 1D-1D-setting. Filled circles indicate form-meaning pairs for sending, open circles for receiving; the colour indicates the reward (red: high; yellow: low); grey lines indicate the corresponding form-meaning pairs of the other agent; heat maps show the expected reward $r(f,m)$ estimated by the agent. A total of 400 out of 10000 communication acts per iteration is shown.



(a) **(b)** **(c)**

Figure 3: 1D-1D-setting with a one-dimensional world (blue) and a one-dimensional form space (orange). **(a):** Optimal mapping as in the final iteration in Figure 2. **(b):** Sub-optimal mapping, where one end of the meaning space is mapped to the "wrong" end of the form space, resulting in a fragmented form space with a cut (dashed line). **(c):** The correspond sub-optimal result in the simulations; misunderstandings at the cut (due to exploratory behaviour) appear as long grey lines.

In our model, forms are transmitted from the sender to the receiver with additional Gaussian noise to better capture the real-world conditions of communication. The communication success is measured by a reward function and after each communication act, the resulting reward is observed by both agents. The reward is maximal for identical meanings m_A and m_B and decays with a Gaussian shape for increasing distance $|m_A - m_B|$. The agents attempt to maximise the reward by adapting their sender and receiver policies. However, each agent only has access to their own form and meaning, not to those of the other agent. This implies that each agent's sender and receiver policy are coupled, making them consistent for each agent individually but not between agents.

Learning is performed using an iterative approach from reinforcement learning, called *policy iteration* (Sutton & Barto, 2018; Lagoudakis & Parr, 2003). In each iteration, the two agents perform 5000 acts of communication in each direction. Each of these 10000 data points specifies the reward ρ that was obtained at a point (f,m) in form-meaning space. From these data, each agent estimates the expected reward using linear regression with Gaussian basis functions (Bishop, 2007; Hastie et al., 2008).

The policies are adjusted to choose forms and meanings with a probability that is proportional to the expected reward. The policies thus do not always choose the form and meaning with the maximum expected reward but, to a lesser extent, also explore sub-optimal choices. This explorative behaviour is required to ensure robust convergence of the policies in reinforcement learning (Sutton & Barto, 2018).

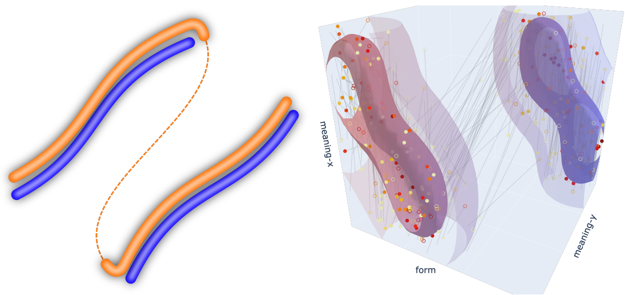
Results and Discussion

We conducted simulations in three different environments. First, we used a simple 1D-1D-setting as illustrated in Figure 3 to illustrate the learning process and investigate its properties, in particular, the influence of different conditions of transmission noise and examples of sub-optimal signalling conventions with a fragmented form space. Second, we investigated the scenario of modal worlds (Figure 4) and showed that the modal structure is reflected in distinct symbols. Third, we performed a simulation with a mismatch in the topology of the form and meaning space (Figure 5), showing that this leads to a discretisation in form space despite a continuous meaning space.

Optimal vs. Sub-Optimal Signalling Conventions

Figure 2 shows a complete learning process in the 1D-1D-setting, from the initial state with broad and unstructured policies to the final state with an optimal one-to-one mapping between forms and meanings. However, the policies did not always converge to an optimal signalling convention: Figure 3(c) shows an example with a fragmented mapping. To better understand the reasons for such a fragmentation we performed an extensive statistical evaluation of the probability of converging to the optimal signalling convention for different conditions of the transmission noise, which provided two central insights.

First, the chances of converging to an optimal signalling convention strongly depend on the amount of noise (variance of the Gaussian) in the transmission distribution with more noise leading to a more reliable convergence. This is in agreement with the observations made by Zuidema & Westermann (2003). Second, sub-optimal signalling conventions with a fragmented form space are *locally* optimal and stable. They



(a) **(b)**

Figure 4: **(a):** Worlds that exhibit distinct modes (blue lines) induce a discretisation in form space (orange), which has to be cut (dashed line) in order to be mapped to the world. **(b):** Results of the simulations, showing single communication acts and the expected reward, as in Figure 2, but in three dimensions. Instead of a heat map, we show isosurfaces at $r(f,m) = 0.5$ (high opacity) and $r(f,m) = 0.05$ (low opacity). An interactive version of the figure is available in the supplementary material.

are highly unlikely to change once being established.

These results constitute a pragmatic reason for the emergence of discretisation in regions with continuous meaning, which is due to locally optimal but globally sub-optimal signalling conventions. Even if an optimal continuous mapping between forms and meanings exists, this optimal solution is not guaranteed to be found.

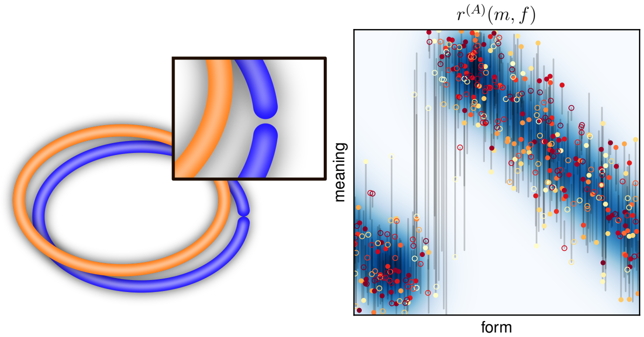
Modal Worlds

In our second simulation, the environment consists of two separate lines (Figure 4) that are embedded in a two-dimensional meaning space. We generated meanings with additional Gaussian noise, so that each one-dimensional line effectively covers an extended region in the two-dimensional meaning space. The form space was a single one-dimensional line as before.

The agents reliably learned an optimal signalling convention with the form space being split in two parts, which were mapped to the two different lines in meaning space, as shown in Figure 4(b). In between the two parts in form space, a gap of forms that were not used in optimal communication emerged. The form space can thus be considered to comprise two distinct symbols, corresponding to the two separate modes of the world, with each symbol exhibiting a continuous (iconic) mapping of its internal form space to the meaning space of the respective mode.

The fact that meanings from a continuous but modal world can be effectively communicated using a discrete signalling system was mathematically investigated by Feldman (2012) and is empirically confirmed by our findings.

Additionally, our simulations demonstrate that discretisation in form space does not need to be assumed in this scenario but instead emerges from a continuous form space as the result of a learning process. Furthermore, the results demonstrate that the learned signalling system exhibits both discrete and continuous properties that allow to not only refer to the separate modes but also represent more fine-grained differences *within* each of the modes. This is structurally similar to how words in a language may be used to denote clearly distinct meanings, while intonation may supplement additional information to communicate these meanings more precisely.



(a) **(b)**

Figure 5: **(a):** Topological mismatch between a one-dimensional world (blue) and a one-dimensional but *circular* form space (orange). Misunderstandings occur at the end points of the meaning space, which are mapped close to the same point in form space (inset). Creating a gap in form space and avoiding symbols within that region reduces misunderstandings. **(b):** Corresponding simulation results with the form space wrapping around horizontally. To facilitate learning, the agents continue exploring the gap region, which results in misunderstandings with a low reward (long grey lines).

Topological Mismatch

In our third experiment, we employed a circular form space and a bounded line as meaning space, as illustrated in Figure 5. This is the simplest case of a mismatch in the topology of the form an meaning space, while both spaces are at the same time inherently continuous.

In the optimal mapping learned by the agents, shown in Figure 5(b), the form space is continuously mapped to the meaning space, except at the boundaries of the meaning space. At this point, two maximally different meanings are mapped close to each other in form space. To avoid misunderstandings, a gap in form emerges, similar to the scenario with two separate lines in meaning space. However, in the present case the meaning space is entirely continuous and the gap only emerges as a consequence of the topological mismatch between form and meaning space.

This topological argument explains the emergence of discretisation on a fundamental and abstract level and it can be expected that any signalling system is characteristically shaped by the topology of the corresponding form and meaning space.

In reality, the relevant topologies are of course much more complex. For instance, the human vowel system has at least three major dimensions (tongue position, tongue height, lip roundedness) and several subordinate dimensions that are not generally independent (Ladefoged & Maddieson, 1990). Beyond language, music has the capacity to convey emotions and other complex states of mind (Juslin & Laukka, 2003). The spaces of musical objects, such as tones, chords or interacting polyphonic voices, have a highly complex topology. For instance, the space of musical keys and triads alone has a topology that can be alternatively described as a planar two-dimensional *Tonnetz* (Euler, 1739; Riemann, 1896), a tube, or a torus, depending on which properties of the tones are considered to be relevant (Cohn, 1997; Krumhansl, 1998; Chew, 2000; Lieck et al., 2020).

These are only two specific examples of non-trivial topologies that arise in the real world. There are many more modes of communication, each with their own topological particularities. It is therefore highly plausible that the topological effects we observe in our experiments also play a role in real-world communication.

Conclusion

We investigated the emergence of discrete symbols by simulating the learning process of two agents that acquire a shared signalling system. We empirically confirmed three causes for discretisation: 1) convergence to sub-optimal signalling conventions with a fragmented form space, 2) modal worlds, as suggested by Feldman (2012), and 3) a topological mismatch between form and meaning space, as conjectured by De Boer & Verhoef (2012) and Zuidema & Westermann (2003).

We observe continuous mappings between forms and meanings for parts of the spaces with a matching topology. These relations are *iconic* in that the form and the meaning space resemble each other in terms of the topology, so that a continuous variation of the form corresponds to an analogous variation in meaning (also cf. De Boer & Verhoef, 2012).

The joint treatment of discrete and continuous properties based on a definition of discrete symbols in continuous form and meaning spaces allows us to model the emergence of discretisation as well as the coexistence, coevolution and interplay of discrete and continuous properties in communication. These aspects are not only relevant to human language but also to other forms of communication, such as music and communication in non-human animals.

References

Bishop, C. M. (2007). *Pattern Recognition and Machine Learning (Information Science and Statistics)* (1st ed. 2006. Corr. 2nd printing ed.). Springer. Hardcover.

Bleys, J., Loetzsch, M., Spranger, M., & Steels, L. (2009). *The grounded colour naming game*. Institute of Electrical and Electronics Engineers. doi: http://dx.doi.org/10.13039/501100000780

Cangelosi, A., & Parisi, D. (Eds.). (2002). *Simulating the evolution of language*. London ; New York: Springer.

Chandler, D. (2017). *Semiotics: The basics* (Third ed.). Taylor & Francis.

Chew, E. (2000). *Towards a mathematical model of tonality*. PhD Thesis, Massachusetts Institute of Technology.

Christiansen, M. H., & Kirby, S. (2003). *Language evolution*. Oxford University Press.

Cohn, R. (1997). Neo-riemannian operations, parsimonious trichords, and their " tonnetz" representations. *Journal of Music Theory*, *41*(1), 1–66.

De Boer, B. (2000). Self-organization in vowel systems. *Journal of phonetics*, *28*(4), 441–465.

De Boer, B. (2012). Iconicity in Structured Form and Meaning Spaces. In *The Evolution Of Language* (pp. 424–425).

De Boer, B., & Verhoef, T. (2012). Language dynamics in structured form and meaning spaces. *Advances in Complex Systems*, *15*(03n04), 1150021.

De Jong, E. D. (1999). Analyzing the Evolution of Communication from a Dynamical System's Perspective. In *European Conference on Artificial Life* (pp. 689–693). Springer.

Dimos, K., Dick, L., & Dellwo, V. (2015). Perception of levels of emotion in speech prosody. *The Scottish Consortium for ICPhS*.

Euler, L. (1739). *Tentamen novae theoriae musicae ex certissimis harmoniae principiis dilucide expositae*. St. Petersburg: Ex Typographia Academiae Scientiarum.

Feldman, J. (2012, April). Symbolic representation of probabilistic worlds. *Cognition*, *123*(1), 61–83. doi: 10.1016/j.cognition.2011.12.008

Hastie, T., Tibshirani, R., & Friedman, J. (2008). *The elements of statistical learning: Data mining, inference and prediction* (Second ed.). Springer.

Janik, V. M., & Slater, P. J. (1997). Vocal learning in mammals. *Advances in the Study of Behaviour*, *26*, 59–100.

Juslin, P. N., & Laukka, P. (2003). Communication of emotions in vocal expression and music performance: Different channels, same code? *Psychological bulletin*, *129*(5), 770.

Krumhansl, C. L. (1998). Perceived triad distance: Evidence supporting the psychological reality of neo-Riemannian transformations. *Journal of Music Theory*, *42*(2), 265–281.

Kuhl, P. K. (2004, November). Early language acquisition: Cracking the speech code. *Nature Reviews Neuroscience*, *5*(11), 831. doi: 10.1038/nrn1533

Ladefoged, P., & Maddieson, I. (1990). Vowels of the world's languages. *Journal of Phonetics*, *18*(2), 93–122.

Lagoudakis, M. G., & Parr, R. (2003, December). Least-squares Policy Iteration. *J. Mach. Learn. Res.*, *4*, 1107–1149.

Lieck, R., Moss, F. C., & Rohrmeier, M. (2020, October). The Tonal Diffusion Model. *Transactions of the International Society for Music Information Retrieval*, *3*(1), 153–164. doi: 10.5334/tismir.46

Little, H., Eryılmaz, K., & de Boer, B. (2017, November). Signal dimensionality and the emergence of combinatorial structure. *Cognition*, *168*, 1–15. doi: 10.1016/j.cognition.2017.06.011

Moulin-Frier, C., Nguyen, S. M., & Oudeyer, P.-Y. (2014). Self-organization of early vocal development in infants and machines: The role of intrinsic motivation. *Frontiers in Psychology*, *4*, 1006.

Moulin-Frier, C., & Oudeyer, P. (2012, November). Curiosity-driven phonetic learning. In *2012 IEEE International Conference on Development and Learning and Epigenetic Robotics (ICDL)* (pp. 1–8). doi: 10.1109/DevLrn.2012.6400583

Murakami, M., Kröger, B., Birkholz, P., & Triesch, J. (2015, August). Seeing [u] aids vocal learning: Babbling and imitation of vowels using a 3D vocal tract model, reinforcement learning, and reservoir computing. In *2015 Joint IEEE International Conference on Development and Learning and Epigenetic Robotics (ICDL-EpiRob)* (pp. 208–213). doi: 10.1109/DEVLRN.2015.7346142

Nolfi, S., & Mirolli, M. (Eds.). (2010). *Evolution of Communication and Language in Embodied Agents*. Berlin, Heidelberg: Springer Berlin Heidelberg. doi: 10.1007/978-3-642-01250-1

Nowak, M. A., & Krakauer, D. C. (1999). The evolution of language. *Proceedings of the National Academy of Sciences*, *96*(14), 8028–8033.

Oliphant, M., & Batali, J. (1997). Learning and the emergence of coordinated communication. *Center for research on language newsletter*, *11*(1), 1–46.

Ouattara, K., Lemasson, A., & Zuberbühler, K. (2009, December). Campbell's monkeys concatenate vocalizations into context-specific call sequences. *Proceedings of the National Academy of Sciences*, *106*(51), 22026–22031. doi: 10.1073/pnas.0908118106

Oudeyer, P.-Y. (2003, July). The production and recognition of emotions in speech: Features and algorithms. *International Journal of Human-Computer Studies*, *59*(1), 157–183. doi: 10.1016/S1071-5819(02)00141-6

Oudeyer, P.-Y. (2005, April). The self-organization of speech sounds. *Journal of Theoretical Biology*, *233*(3), 435–449. doi: 10.1016/j.jtbi.2004.10.025

Oudeyer, P.-Y., & Kaplan, F. (2007, March). Language evolution as a Darwinian process: Computational studies. *Cognitive Processing*, *8*(1), 21–35. doi: 10.1007/s10339-006-0158-3

Rendell, L., & Whitehead, H. (2001, April). Culture in whales and dolphins. *Behavioral and Brain Sciences*, *24*(2), 309–324. doi: 10.1017/S0140525X0100396X

Riemann, H. (1896). *Dictionary of music* (J. S. Shedlock, Trans.). London: Augener.

Roberson, D., Davidoff, J., Davies, I. R. L., & Shapiro, L. R. (2004). The Development of Color Categories in Two Languages: A Longitudinal Study. *Journal of Experimental Psychology: General*, *133*(4), 554–571. doi: 10.1037/0096-3445.133.4.554

Sandhofer, C. M., & Smith, L. B. (1999). Learning Color Words Involves Learning a System of Mappings. *Developmental Psychology*, *35*(3), 668–679.

Scherer, K. R. (2003, April). Vocal communication of emotion: A review of research paradigms. *Speech Communication*, *40*(1), 227–256. doi: 10.1016/S0167-6393(02)00084-5

Schröder, M. (2001). Emotional speech synthesis: A review. In *Seventh European Conference on Speech Communication and Technology.*

Spranger, M. (2016). *The evolution of grounded spatial language*. Language Science Press.

Steels, L. (1997). The synthetic modeling of language origins. *Evolution of communication*, *1*(1), 1–34.

Steels, L., & Belpaeme, T. (2005). Coordinating perceptually grounded categories through language: A case study for colour. *Behavioral and brain sciences*, *28*(4), 469–488.

Sutton, R. S., & Barto, A. G. (2018). *Reinforcement learning: An introduction* (Second edition ed.). Cambridge, Massachusetts: The MIT Press.

Wilbrecht, L., & Nottebohm, F. (2003). Vocal learning in birds and humans. *Mental Retardation and Developmental Disabilities Research Reviews*, *9*(3), 135–148. doi: 10.1002/mrdd.10073

Wittgenstein, L. (1953). *Philosophical investigations. Philosophische Untersuchungen.*

Zuidema, W., & Westermann, G. (2003). Evolution of an optimal lexicon under constraints from embodiment. *Artificial Life*, *9*(4), 387–402.